Watermarking Polygonal Lines Using an Optimal Detector On the Fourier Descriptors Domain

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Introduction

❖ **Polygonal Line: ordered set of vertices, connected by straight traces that describe an open or a closed line. Polygonal lines are present in GIS cartographies, vector graphics or contour descriptions.**

❖ **Digital Watermarking: Information embedding in digital material, usually to stamp a copyright mark.**

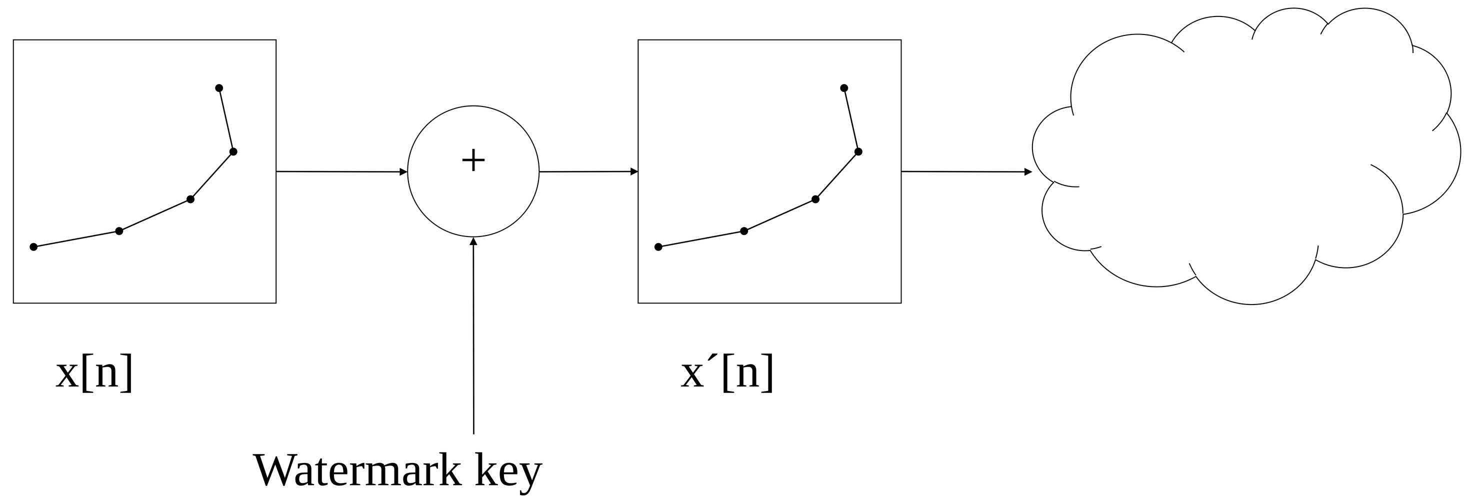
❖ **Watermarking a Polygonal Line consists on slightly modifying the vertices of the line, so that the polygonal line does not suffer essential changes while carrying some extra information.**

❖ **Contour watermarking is achieved by modifying the Fourier Descriptors magnitude in an imperceptible way.**

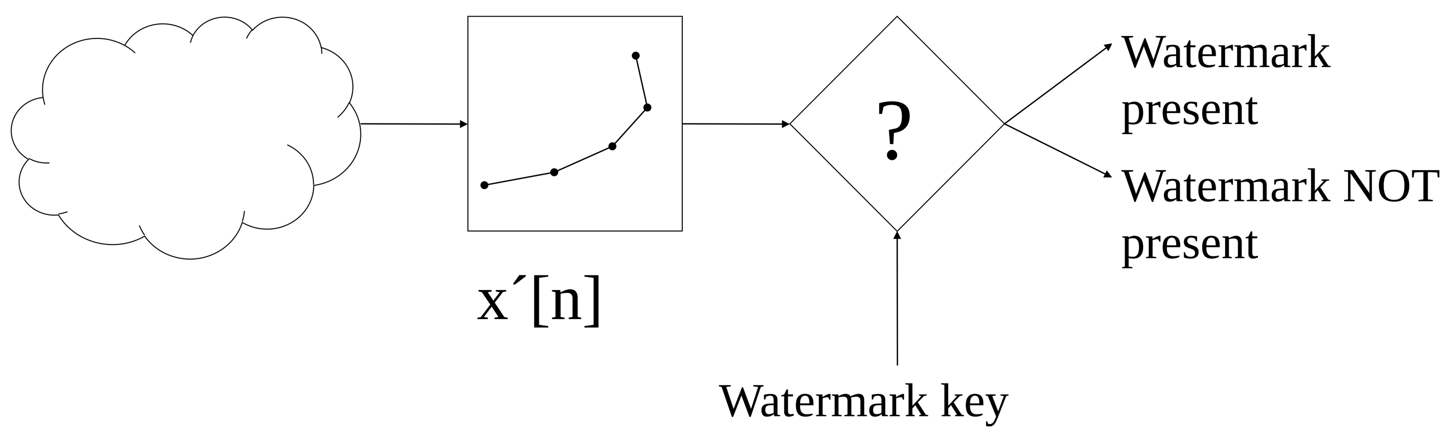
❖ **Robustness is a desired property of the watermark that consists on the difficulty to remove that extra information from the Polygonal Line, on despite of intentional or unintentional manipulation of the line.**

❖ **Watermarks generated by this technique can be successfully detected even after rotation, translation, scaling or reflection of the host polygonal line.**

Watermark embedding



Watermark detection



Watermark embedding

❖ Aimed at imperceptibly embedding a secret key in the polygonal line.

1) The *Polygonal Line* is considered as an ordered set of points in the complex space.

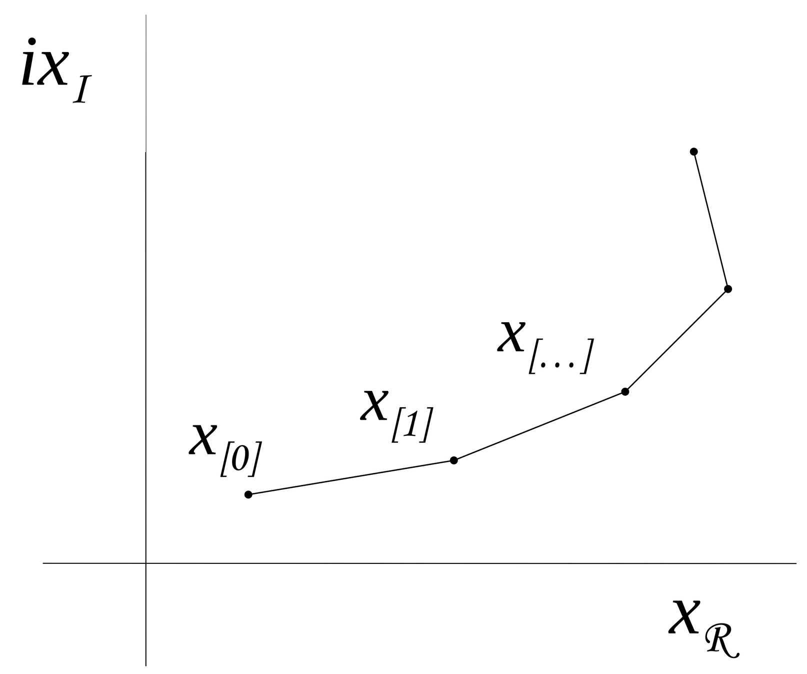


Fig. 1. Polygonal Line

2) The *Fourier Descriptors* are the coefficients of the DFT performed on x[n]:

$$\mathbf{X[k]} = \sum_{n=0}^{N-1} x[n]\exp\left(\frac{-i2kn\pi}{N}\right), 0 \le k \le N-1$$

3) The *Watermark sequence* is a binary pseudorandom sequence taking the value ± 1. The sequence is generated from a secret key.

4) The watermark is embedded in the Fourier Descriptros according to the multiplicative formula:

$$|X'[k]| = |X[k]|(1+sW[k]), k = 0,1,\ldots, N-1$$

5) An inverse DFT is performed to obtain the coefficients x´[n] of the watermarked line

Watermark detection

❖ Aimed at determining whether a secret key is present or not in a polygonal line x[n].

❖ The simplest approach is to calculate the **correlation** of the line with the watermark:

$$C = \frac{1}{N}\sum_{k=0}^{N-1}|X[k]|W[k]$$

❖ The correlation value C is compared against a threshold T.

❖ The statistical detection theory states that the correlation is the optimal signal (watermark) detector if the noise (host signal) is additive and the noise samples are independent random variables following a Gaussian distribution. These assumptions do not hold and the correlator detector can be improved.

Optimal detection

❖ The ***Likelihood Ratio Test*** compares the conditional probabilities of the polygonal line **X** to belong to the hypothesis of watermark presence (H_1) or absence (H_0):

$$\Lambda = \frac{p(\mathrm{X}|H_1)}{p(\mathrm{X}|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} T$$

❖ If the samples X[k] are assumed to be independent, then:

$$p(\mathrm{X}|H_j) = \prod_{k=0}^{N-1} p(X[k]|H_j), j = 0,1$$

❖It is supposed that $p(X[k]|H_0)$ follows a known probability distribution function *f*. As the watermark embedded is a linear operation, the conditional probability $p(X[k]|H_1)$ can also be expressed in terms of *f*. Thus, the Likelihood Ratio Test is:

$$\Lambda = \frac{\prod_{k=0}^{N-1} f\left(X[k](1+sW[k])\right)}{\prod_{k=0}^{N-1}(1+s)f\left(X[k]\right)} \underset{H_0}{\overset{H_1}{\gtrless}} T$$

❖ The probability distribution function *f* is supposed to follow a Raileigh distribution :

$$p(X[k]) = \frac{|X[k]|}{\sigma_k^2}\exp\left(-\frac{|X[k]|^2}{2\sigma_k^2}\right)$$

❖ Mean and variance are estimated from the signal **X**, and are assumed constant at least for small intervals of *k*.

Properties

❖ Due to the properties of the Fourier Descriptors, a watermarked polygonal line keeps the watermark even if has suffered from several geometrical distortions.

- ❖ Rotation
- ❖ Scaling
- ❖Translation
- ❖Change of traversal starting vertex
- ❖Inversoin of traversal direction
- ❖Mirroring

Conclusions

❖ This is a watermarking system for polygonal lines (application in GIS, vector graphics, SVG etc.)

❖ Any polygonal line with more than 1000 points is suitable for being watermarked. As statistical methods are used, the higher the number of points, the better the reliability.

❖ More work on real multiple-line images should be done.

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